**Finding Redundant and Complementary Communities in Multidimensional Networks**

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### Problem

**Problem 1 (MCD)** Given a multidimensional network \( G \), find and characterize the multidimensional communities.

### Characterization

Examples of multidimensional communities

- Variety \( V_c \): # of different dimensions
- Exclusivity \( E_c \): # of pairs connected by only one dimension
- Homogeneity \( H_c \): distribution of edges over dimensions

We aggregate the above by their product:

\[
\gamma_c = V_c \times E_c \times H_c
\]

### Algorithm

**Algorithm 1 MCD_Solver**

**Require:** \( G, \phi, CD \)

**Ensure:** set of multidimensional communities \( C \) and sets of their characterization \( S_c, S_\phi \)

1. \( G \leftarrow \phi(G) \)
2. \( C \leftarrow CD(G) \)
3. for all \( c \in C \) do
4. \( c \leftarrow \phi(c) \)
5. \( C \leftarrow C \cup c \)
6. \( S_\phi \leftarrow S_\phi \cup \mu(c) \)
7. \( S_S \leftarrow S_S \cup \gamma(c) \)
8. end for
9. return \( C, \Gamma, P \)

### Mapping Function \( \phi \)

We use three different \( \phi \):

- Connectivity check:
  \[
  \mu_{u,v} = \begin{cases} 
  1 & \text{if } \exists \{d : (u, v, d) \in E\} \\
  0 & \text{otherwise}
  \end{cases}
  \]

- Number of dimensions:
  \[
  \nu_{u,v} = |\{d : (u, v, d) \in E\}|
  \]

- Multidimensional clustering coefficient:
  \[
  \eta_{u,v} = 1 + \frac{|N_u \cap N_v|}{|N_u \cup N_v| - 1}
  \]

### Run Through Example

Run through example for three instances of MCD_Solver varying the \( \phi \) parameter

### References